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# EINLADUNG

zum Gastvortrag

von

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am

## Freitag, 22.03.2013, 10.00 (c.t.) Uhr

Technische Universität Wien, Karlsplatz 13, 1040 Wien
**HS 13 Ernst Melan (**Stiege 7, 2. Obergeschoß)

## Energy-Inconsistent Objective Stress Rates in ABAQUS, ANSYS, LS-DYNA and Other FE Codes: Magnitude of Errors and How to Correct Them

The objective stress rates used in most commercial finite element (FE) codes are the Jaumann rate of Cauchy (true) stress for the incremental (updated Lagrangian or Riks) implicit analysis and the Green-Naghdi rate for explicit analysis. However, these two rates cause an error in energy, as they are not work-conjugate to any finite strain tensor except if the material is incompressible or rotations not too lagre. For bifurcation load analysis, most programs use the Jaumann rate of Kirchhoff stress, which is work-conjugate to the Hencky (or logarithmic) stress tensor. Yet this rate causes another kind of energy error in the case of highly orthotropic structures very soft in shear, which require the use of the Truesdell stress rate which is work-conjugate to the Green-Lagrangian finite strain tensor. Although these errors were identified long ego, they were generally thought to be negligible. It is now demonstrated by numerical examples that this is not always so.

First, an example of indentation of a naval-type sandwich plate with a polymeric foam core demonstrates an error of 28.7% in the indentation force and 15.0% in the work on indentation. Similar errors must be expected for all highly compressible materials, such as metallic and ceramic foams, honeycomb, loess, silt, organic soils, pumice, tuff, corral, light wood, osteoporotic bone and various biologic tissues. As a remedy, it is shown that a previously derived equation relating the tangential moduli tensors associated with the Jaumann rates of Cauchy and Kirchhoff stresses can be used in the user's material subroutine of a black-box commercial program to cancel the error due the lack of work-conjugacy and force the program to give results corresponding to the Jaumann rate of Kirchhoff stress, which is work-conjugate.

Second, it is shown that, in buckling of compressed highly orthotropic columns or sandwich columns very soft in shear, the use of Jaumann stress rates can cause large errors, as high as 100% of the critical load, even if the strains are small. Another example of a highly orthotropic plate, equivalent to a large rectangular foam-core sandwich plate for the cladding of a steel ribbed hull of a large ship, shows a critical load error of 40%. Similar situations could arise when severe damage such as distributed cracking leads to a highly anisotropic tangential stiffness matrix, or when axial cracks between fibers severely weaken a uniaxial fiber composite. It is demonstrated that if a certain previously derived stress-dependent transformation of the tangential moduli is introduced into the user’s material subroutine in these commercial programs, they will yield correct results that correspond to Truesdell’s stress rate and are also supported by experiments. These corrections, however, introduce a numerical error which can be minimized by sufficiently small load steps

Finally, an example of plastic shear in compression is used to show that the Green-Naghdi rate, which employs a material rotation tensor different from the spin tensor, can give an error >100% when the material rotation exceeds about $25^o$.

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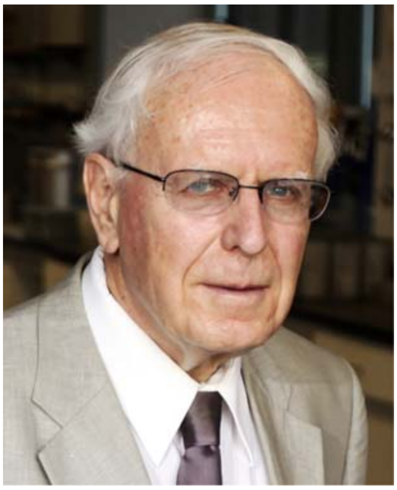

BIO-SKETCH: Born and educated in Prague (Ph.D. 1963), Bažant joined Northwestern in 1969, where he has been W.P. Murphy Professor since 1990 and simultaneously McCormick Institute Professor since 2002, and Director of Center for Geomaterials (1981-87). He was inducted to Nat. Academy of Sciences, Nat. Academy of Engrg. and Am. Acad. of Arts & Aci., as well as Italian Nat. Acad. (dei Lincei), Austrian Acad. of Sciences, Czech Acad. of Engrg., Spanish Royal Acad. of Eng., Eur. Acad. of Sci. & Arts, and Istituto Lombardo. He is an Hon. Member of ASME, ASCE and ACI, and Illinois Registered Structural Engineer. He received 7 honorary doctorates (Prague, Karlsruhe, Colorado, Milan, Lyon, Vienna, Ohio State), ASME Timoshenko, Warner and Nadai Medals, ASCE von Karman, Newmark, Biot and Croes Medals and Lifetime Achievement Award, SES Prager Medal, RILEM L'Hermite Medal, Torroja Medal, etc. He authored six books: Scaling of Structural Strength, Inelastic Analysis, Fracture and Size Effect, Stability of Structures, Concrete at High Temperatures, and Concrete Creep. With H-index 77 and >24,600 citations on Google (minus ~3000 self), he is one of the original top 100 ISI Highly Cited Scientists in engineering (www.ISIhighlycited.com). http://cee.northwestern.edu/people/bazant/